

Modulation effect on the spin Hall resonance

SK Firoz Islam

Department of Physics, Indian Institute of Technology-Kanpur, Kanpur-208 016, India

(Dated: January 22, 2013)

The effect of a weak electrical modulation on spin Hall resonance is presented here. In presence of the magnetic field normal to the plane of the motion of electron, the Landau levels are formed which get broadened due to the weak modulation. The width of the Landau levels broadening are periodic with the inverse magnetic field. There is a certain magnetic field for which the crossing of Landau levels between spin-up and spin-down branches takes place. This gives rise to the resonance in the spin Hall conductivity (SHC). The Landau levels broadening or the energy correction due to the modulation removes the singularity appears at the resonance field in SHC, leading to the suppression of SHC accompanied by two new peaks around this point. The separation of these two peaks increases with the increase of the modulation period. Moreover, we find that the height of the two peaks are also modulation period dependent.

PACS numbers: 71.70.Ej, 73.43.Qt, 85.75.-d

I. INTRODUCTION

The lack of structural symmetry in a two dimensional electron gas (2DEG) has attracted lots of attention of condensed matter physicists due to the possible remarkable future applications as proposed by Datta and Das¹⁻⁴. Various experiments have confirmed the existence of a spin-orbit coupling known as Rashba spin-orbit coupling (RSOC) in a 2DEG with structural inverse asymmetry⁵. The RSOC lifts the spin degeneracy of electron even in absence of any magnetic field and provide a nice way to manipulate electron's spin degree of freedom. Moreover, the RSOC is responsible for some noble effects like spin-FET^{1,6}, metal-insulator transition in a two-dimensional hole gas⁷, spin-resolved ballistic transport⁸ and spin-galvanic effect⁹. The experimental confirmation of the enhancement of RSOC strength up to a significant amount by the application of the gate voltage has made this field much more attractive^{10,11}. The RSOC produces two different energy branches for spin-up and spin-down electrons^{12,13} and manifests itself through transport^{5,14,15}, magnetic¹⁶ and optical properties¹⁷. The spin Hall effect (SHE)^{18,19} is one of the most unique property exhibits by 2DEG with RSOC. The SHE has been studied extensively theoretically after the experimental detection of the spin accumulation on opposite sides of the sample with opposite polarization^{20,21}. In presence of magnetic field, an additional degeneracy comes into picture when Landau levels of two opposite branches overlap for a certain magnetic field. This degeneracy is the outcome of the competition between Zeeman splitting and Rashba splitting. This additional degeneracy manifests through a jump in the spin polarization and the resonance in SHC²². Later, the effect of disorder in the spin Hall resonance has been studied theoretically and showed the appearance of double peaks around the resonance field instead of single peak at the resonance point²³. The splitting of single peak into double is the effect of the disorder induced Landau levels broadening.

The disorder is not the only sole way to broaden the width of Landau levels. The Landau levels can be broadened by applying a weak electric or magnetic modulation^{24,25}. The width of the Landau levels due to the modulation is periodic with inverse magnetic field whereas the disorder induced broadening is magnetic field independent. This kind of modulation induced oscillatory broadening of Landau levels results in oscillatory diffusive conductivity in inverse magnetic field, which is known as Weiss oscillation^{24,25}. Moreover, the modulation effect on the spin Hall conductivity has been studied theoretically also and shows the Weiss type oscillation accompanied by beating in SHC at low magnetic field²⁶. In this paper, our major aim is to study the effect of modulated Landau levels in spin Hall resonance.

This paper is organized as follows. In section II, we summarize the exact energy eigenvalues and the corresponding eigenfunctions of a 2DEG with the Rashba SOI in presence of a perpendicular uniform magnetic field. The formalism is given in section III for the calculation of SHC. Numerical results are presented in section IV. We present a summary of our work in Sec. V.

II. ENERGY SPECTRUM AND ENERGY CORRECTION DUE TO THE MODULATION IN PRESENCE OF THE RASHBA SOI

The Hamiltonian of an electron ($-e$) with the RSOC in presence of a perpendicular magnetic field $\mathbf{B} = B\hat{z}$ is given by

$$H = \frac{(\mathbf{p} + e\mathbf{A})^2}{2m^*} \sigma_0 + \frac{\alpha}{\hbar} [\boldsymbol{\sigma} \times (\mathbf{p} + e\mathbf{A})]_z + \frac{1}{2} g \mu_B B \sigma_z, \quad (1)$$

where \mathbf{p} is the two dimensional momentum operator, m^* is the effective mass of the electron, g is the Lande-factor, μ_B is the Bohr magneton, σ_0 is the unit matrix, $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ are the Pauli spin matrices, and α is the strength of the RSOC.

Here, we shall just mention the exact solutions of the Hamiltonian H . Using the Landau wave functions without the RSOC as the basis, one can obtain the energy spectrum and the corresponding eigenfunctions¹⁴. For $n = 0$ there is only one level, the same as the lowest Landau level without RSOC, with energy $E_0^+ = E_0 = (\hbar\omega - g\mu_B B)/2$ and the corresponding wave function is

$$\Psi_0^+(k_y) = \frac{e^{ik_y y}}{\sqrt{L_y}} \phi_0(x + x_0) \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \quad (2)$$

Here, $\omega = eB/m^*$, $x_0 = k_y l_0^2$ with $l_0 = \sqrt{\hbar/(eB)}$ is the magnetic length scale. For $n = 1, 2, 3, \dots$ there are two branches of the energy levels, denoted by $+$ corresponding to the "spin-up" electrons and $-$ corresponding to the "spin-down" electrons with energies

$$E_{ns} = n\hbar\omega + s\sqrt{E_0^2 + nE_\alpha\hbar\omega}, \quad (3)$$

where $s = \pm$ and $E_\alpha = 2m^*\alpha^2/\hbar^2$ is the Rashba energy determined by the SOI strength α . The corresponding wave function for $+$ branch is

$$\Psi_n^+(k_y) = \frac{e^{ik_y y}}{\sqrt{L_y A_n}} \begin{pmatrix} D_n \phi_{n-1}(x + x_0) \\ \phi_n(x + x_0) \end{pmatrix}, \quad (4)$$

and the $-$ branch is

$$\Psi_n^-(k_y) = \frac{e^{ik_y y}}{\sqrt{L_y A_n}} \begin{pmatrix} \phi_{n-1}(x + x_0) \\ -D_n \phi_n(x + x_0) \end{pmatrix}, \quad (5)$$

where $A_n = 1 + D_n^2$ and $D_n = \sqrt{nE_\alpha\hbar\omega/[E_0 + \sqrt{E_0^2 + nE_\alpha\hbar\omega}]}$. Here, $\phi_n(x) = (1/\sqrt{\sqrt{\pi}2^n n! l_0}) e^{-x^2/2l_0^2} H_n(x/l_0)$ is the normalized harmonic oscillator wave function with n is the Landau level index,

The condition of the crossing of the energy levels between E_n^+ and E_{n+1}^- can be obtained from $E_n^+ - E_{n+1}^- = 0$, which gives²²

$$\sqrt{(1 - g^*)^2 + \eta n} + \sqrt{(1 - g^*)^2 + \eta(n + 1)} = 2, \quad (6)$$

where $\eta = 4E_\alpha/(\hbar\omega)$, $g^* = gm^*/(2m_0)$ and the quantum number n follows $2n \leq \nu \leq 2(n + 1)$ with $\nu = N_e/N_\phi$ being the filling fraction. Here, N_ϕ is the magnetic flux quanta. The magnetic field at which the above condition satisfy is termed as the resonance point.

Now a weak spatial electric modulation $V = V_0 \cos(Kx)$ is applied to the system along the x -direction with periodicity a , where $K = 2\pi/a$. The first-order energy correction due to this modulation can be easily derived¹⁵ as $\Delta E_{n,k_y,s} = \frac{V_u}{A_n} F_{n,s}$, where $V_u = V_0 \exp(-u/2) \cos(Kx_0)$ with $u = K^2 l_0^2/2$, $F_{n,+}(u) = D_n^2 L_{n-1}(u) + L_n(u)$ and $F_{n,-}(u) = D_n^2 L_n(u) + L_{n-1}(u)$. Here, $L_n(u)$ is the Laguerre polynomial of order n .

III. FORMALISM

We consider a Rashba coupled 2DEG in a $x - y$ plane with size $L_x \times L_y$. In presence of a weak electric field along the x -direction, the electron with opposite spin scattered towards y -axis of the sample and gives rise to SHC. To calculate the SHC, we shall use the Kubo formula as

$$\sigma_{xy}^{\text{SHC}} = -\frac{e\hbar}{L_x L_y} \sum_{\{n,s\} \neq \{n',s'\}, k_y} [f(E_{n,s}) - f(E_{n',s'})] \times \frac{\text{Im} \langle n, k, s | J_y^{S_z} | n', k, s' \rangle \langle n', k, s' | v_x | n, k, s \rangle}{(E_{ns} - E_{n's'})^2 + \delta^2} \quad (7)$$

with the z -component of spin current operator along y -direction $J_y^{S_z} = \frac{1}{2} \{v_y, S_z\}$ as per the conventional definition. Here, δ is assigned to control the height of the resonance peak. The velocity operators are

$$v_x = \frac{p_x}{m^*} \sigma_0 - \frac{\alpha}{\hbar} \sigma_y, \quad (8)$$

$$v_y = (x + x_c) \omega_c \sigma_0 + \frac{\alpha}{\hbar} \sigma_z, \quad (9)$$

and $f(E_{n,s})$ is the equilibrium Fermi distribution function. As per the restriction imposed over summation in the expression of SHC there is no intra-band ($\{n, s\} = \{n', s'\}$) contribution.

For $n' = n + 1$, the matrix elements of spin current operator and velocity operator are

$$\langle n, k, + | J_y^{S_z} | n + 1, k, - \rangle = \frac{1}{\sqrt{A_n A_{n+1}}} \frac{\hbar\omega}{2} \frac{l_0}{\sqrt{2}} \times [D_n \sqrt{n} + D_{n+1} \sqrt{n+1}] \quad (10)$$

and

$$\langle n + 1, k, - | v_x | n, k, + \rangle = i \frac{1}{\sqrt{A_n A_{n+1}}} \frac{\hbar}{\sqrt{2} m^* l_0} \times [D_n \sqrt{n} - D_{n+1} \sqrt{n+1} + \sqrt{2}\zeta], \quad (11)$$

where $\zeta = \sqrt{2} \frac{\alpha m^* l_0}{\hbar^2}$. In presence of modulation the SHC expression need to be modified by including the first order energy correction as $E_{ns}^m \simeq E_{n,s} + \Delta E_{n,k_y,s}$. Now substituting the above matrix elements in Eq. (7) the SHC of modulated system is given by

$$\sigma_{xy}^{\text{SHC}} = -\frac{e}{8\pi} \frac{l_0^2}{a} \int_0^{a/l_0^2} dk_y \sum_n [f(E_{n,+}^m) - f(E_{n+1,-}^m)] \times \frac{\lambda_{n,\zeta}}{[(E_{n,k,+}^m - E_{n+1,k,-}^m)/\hbar\omega]^2} \quad (12)$$

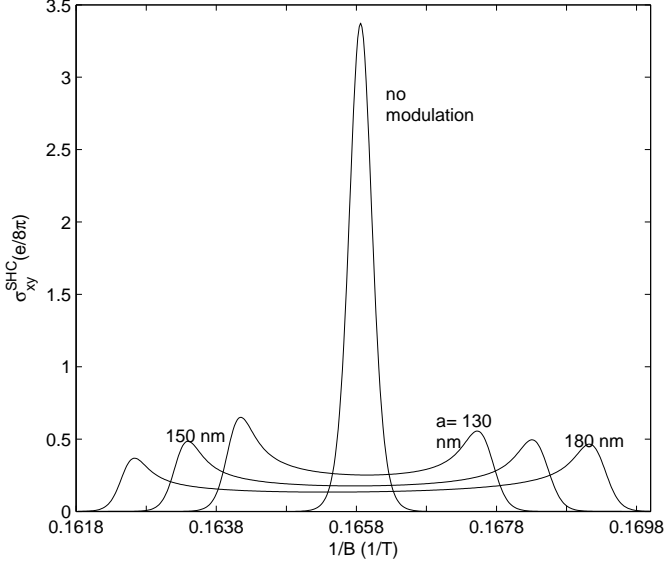


FIG. 1. Plots of the SHC vs inverse magnetic field B for different modulation period. Modulation strength $V_0 = 3$ meV, the temperature $T = 0.6$ K

whereas

$$\lambda_{n,\zeta} = \frac{1}{A_n A_{n+1}} [D_n^2 n - D_{n+1}^2 (n+1) + \sqrt{2}\zeta (D_n \sqrt{n} + D_{n+1} \sqrt{n+1})]. \quad (13)$$

Here, the modulation induced broadening will be acting to reduce the possibility of the occurrence of resonance.

IV. RESULTS AND ANALYSIS

For the numerical calculation, we use the following parameters: electron effective mass $m^* = 0.05m_e$ with m_e as the free electron mass, RSOC strength $\alpha = 0.9\alpha_0$ with $\alpha_0 = 10^{-11}$ eV-m and the carrier concentration $n_e = 1.9 \times 10^{16}/m^2$. These parameters are same as used in several theoretical works for which the resonance appears around $B \simeq 6.1$ T.

We plot the SHC with inverse magnetic field around the resonance point for different modulation periods in Fig.1. It shows that the inclusion of modulation splits the resonance peak into two weak peaks symmetrically on both sides of the resonance point. The physical origin of the appearance of double peaks due to the Landau levels broadening has been discussed in Ref²³. However, in our case the Landau level's broadening is due to the modulation instead of disorder. The resonance point now shows a minimum in conductivity and maintains it for a wide range of magnetic field. The gap between two new peaks which appears on both sides of the resonance field are modulation period dependent. The gap is increasing with

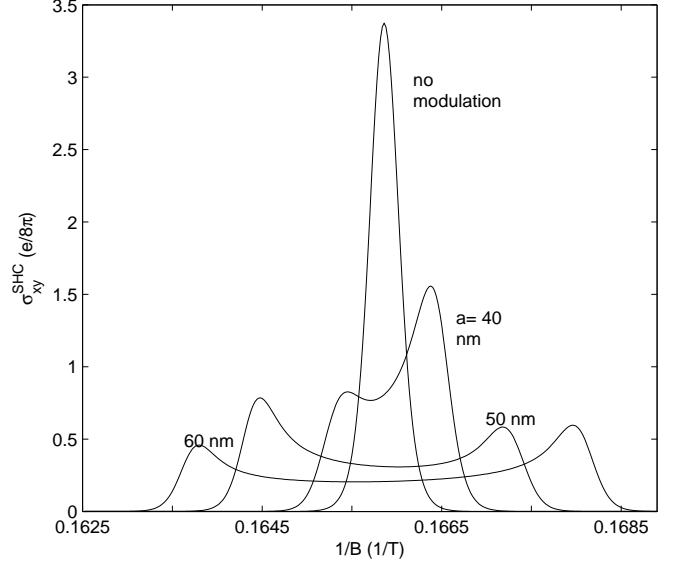


FIG. 2. Plots of the SHC vs inverse magnetic field B for short range modulation period.

increasing modulation period. Note that two new peaks are not confined within the central peak which was observed in the disorder induced Landau broadening case²³. Another point is that the intensity of the two new peaks are decreasing with the increase of the modulation period which can be explained from the expression of energy correction as follows: $\Delta E_{n,k_y,s} \propto V_u = V_0 \exp(-u/2) = V_0 \exp\{-(\pi l_0/a)^2\}$ i.e; the increasing modulation period enhances the effective modulation strength.

In Fig.2, we plot the same but for shorter range of modulation periods. This shows the disappearing of double peaks with decreasing the modulation periods. It is because of the reduction of the effective modulation strength with decreasing periods. Finally, for sufficiently low periodicity two peaks collapsed to a single peak. Fig.3 shows the SHC response for different strength of modulation. The increasing of modulation strength results in suppressing the SHC more. At the same time the flat band between two weak peaks also increases with the increase of the modulation strength. The SHC at the resonance point is decreasing with the increase of the strength of modulation.

In our case, the Landau levels broadening is oscillatory with modulation periods as well as with inverse magnetic field. At Fermi energy, it's oscillation period²⁷ is $2\hbar(k_F \mp k_\alpha)/(eaB)$ with k_F the Fermi vector and $k_\alpha = m^*\alpha^2/\hbar^2$. The level width can be tuned by changing a . This width modulation can not be captured in the small range of magnetic field which is actually taken here around the resonance point. The changing periods results itself only through the suppressing or enhancing the intensity of the double peaks and by widening or

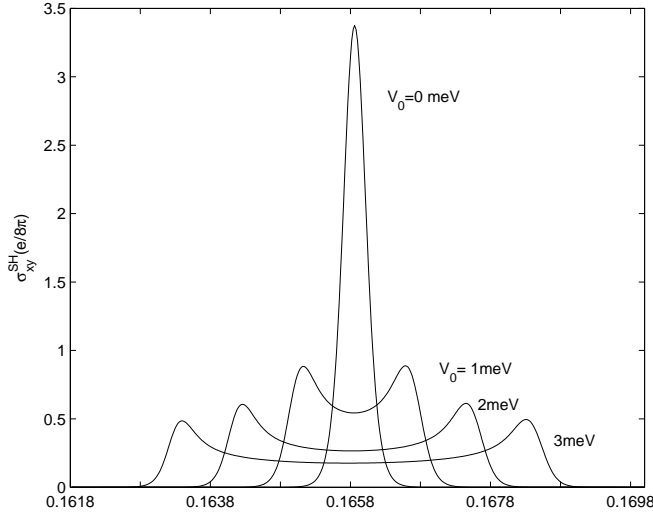


FIG. 3. Plots of the SHC vs different strength of the modulation for $a = 500$ nm.

shortening the gap between these peaks.

V. SUMMARY

We study the effect of modulation induced Landau levels broadening in spin Hall resonance. We find that this level broadening suppress the resonance SHC and splits the central resonance peak into double peaks but relatively weak in strength. The separation between two new peaks can be controlled by the modulation period i.e; the gap increases with increasing modulation period. Moreover, with shortening the modulation periods the intensity of the two new peaks starts to differ. A wide flat band is observed between two peaks in our case whereas in disorder induced level broadening there is a narrow region between two peaks. The increasing strength of modulation reduces the intensity of double peaks and widens the flat band between two peaks.

VI. ACKNOWLEDGEMENT

I would like to thank to Dr. T. K. Ghosh for useful discussions. This work is financially supported by the CSIR, Govt. of India under the grant CSIR-SRF-09/092(0687) 2009/EMR F-O746.

-
- ¹ S. Datta and B. Das, Appl. Phys. Lett. **56**, 665 (1990).
 - ² I. Zutic, J. Fabian, and S. Das Sarma, Rev. Mod. Phys. **76**, 323 (2004).
 - ³ S. A. Wolf, D. D. Awschalom, R. A. Buhrman, J. M. Daughton, S. von Molnr, M. L. Roukes, A. Y. Chtchelkanova, and D. M. Treger, Science **294**, 1488 (2002).
 - ⁴ D. D. Awschalom and M. E. Flatte, Nature Phys **3**, 153 (2007).
 - ⁵ B. Das, D. C. Miller, S. Datta, R. Reifenberger, W. P. Hong, P. K. Bhattachariya, J. Sing, and M. Jaffe, Phys. Rev. B **39**, 1411 (1989).
 - ⁶ S. Bandyopadhyay and M. Cahay, Appl. Phys. Lett. **85**, 1814 (2004).
 - ⁷ E. Tutuc, E. P. De Poortere, S. J. Papadakis, and M. Shayegan, Phys. Rev. Lett. **86**, 2858 (2001); S. J. Papadakis, E. P. De Poortere, H. C. Manoharan, M. Shayegan, and R. Winkler, Science **283**, 2056 (1999).
 - ⁸ J. P. Lu, J. B. Yau, S. P. Shukla, M. Shayegan, L. Wisinger, U. Rossler, and R. Winkler, Phys. Rev. Lett. **81**, 1282 (1998).
 - ⁹ S. D. Ganichev, E. L. Ivchenko, V. V. Bel'kov, S. A. Tarasenko, M. Sollinger, D. Weiss, W. Wegscheider, and W. Pretz, Nature (London) **417**, 153 (2002).
 - ¹⁰ J. Nitta, T. Akazaki, H. Takayanagi, and T. Enoki, Phys. Rev. Lett. **78**, 1335 (1997).
 - ¹¹ T. Matsuyama, R. Kursten, C. Meibner, and U. Merkt, Phys. Rev. B **61**, 15588 (2000).
 - ¹² E. I. Rashba and V. I. Sheka, Dokl. Akad. Nauk SSSR **2**, 162 (1959); E. I. Rashba, Sov. Phys. Solid State **2**, 1109 (1960).
 - ¹³ Y. A. Bychkov and E. I. Rashba, J. Phys. C: Solid State **17**, 580 (1984).
 - ¹⁴ X. F. Wang and P. Vasilopoulos, Phys. Rev. B **67**, 085313 (2003).
 - ¹⁵ X. F. Wang, P. Vasilopoulos, and F. M. Peeters, Phys. Rev. B **71**, 125301 (2005).
 - ¹⁶ Z. Wang, W. Zhang, and P. Zhang, Phys. Rev. B **79**, 235327 (2009).
 - ¹⁷ D. W. Yuan, W. Xu, Z. Zeng, and F. Lu, Phys. Rev. B **72**, 033320, (2005).
 - ¹⁸ M. I. Dyakonov and V. I. Perel', JETP Lett. **13**, 467 (1971).
 - ¹⁹ S. Murakami, N. Nagaosa, and S. C. Zhang, Science **301**, 1348 (2003).
 - ²⁰ Y. K. Kato, R. C. Myers, A. C. Gossard, and D. D. Awschalom, Science. **306**, 1910 (2004).
 - ²¹ J. Wunderlich, B. Kaestner, J. Sinova, T. Jungwirth, Phys. Rev. Lett. **94**, 047204 (2005).
 - ²² S. Shen, M. Ma, X. C. Xie and F. C. Zhang, Phys. Rev. Lett. **92**, 256603 (2004).
 - ²³ Z. Jiang, S. Shen, and F. Zhang, Phys. Rev. B **80**, 195301 (2009).
 - ²⁴ F. M. Peeters and P. Vasilopoulos, Phys. Rev. Lett. **63**, 2120 (1989).
 - ²⁵ F. M. Peeters and P. Vasilopoulos, Phys. Rev. B **47**, 1466 (1993).
 - ²⁶ X. F. Wang and P. Vasilopoulos, Phys. Rev. B **75**, 075331 (2003).
 - ²⁷ SK Firoz Islam and Tarun Kanti Ghosh, J. Phys.: Condens. Matter **24**, 185303 (2012).